2 Waves

2.1: Waves Motion

General equation of wave: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.

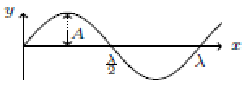
Notation: Amplitude A, Frequency ν, Wavelength λ, Period T, Angular Frequency ω, Wave Number k,

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}, \quad v = \nu\lambda, \quad k = \frac{2\pi}{\lambda}$$

Progressive wave travelling with speed v:

$$y = f(t - x/v), \rightsquigarrow +x; \quad y = f(t + x/v), \rightsquigarrow -x$$

Progressive sine wave:



$$y = A\sin(kx - \omega t) = A\sin(2\pi\,(x/\lambda - t/T))$$

2.2: Waves on a String

Speed of waves on a string with mass per unit length μ and tension T: $v = \sqrt{T/\mu}$

Transmitted power: $P_{\text{av}} = 2\pi^2 \mu v A^2 \nu^2$

Interference:

$$y_1 = A_1 \sin(kx - \omega t), \quad y_2 = A_2 \sin(kx - \omega t + \delta)$$

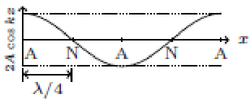
$$\boxed{y = y_1 + y_2 = A\sin(kx - \omega t + \epsilon)}$$

$$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\delta}$$

$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive.} \end{cases}$$

Standing Waves:

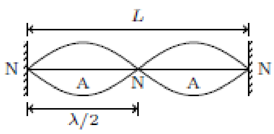


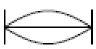
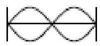
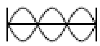
$$y_1 = A_1 \sin(kx - \omega t), \quad y_2 = A_2 \sin(kx + \omega t)$$

$$\boxed{y = y_1 + y_2 = (2A\cos kx)\sin\omega t}$$

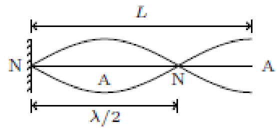
$$x = \begin{cases} \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, & \text{nodes;} \quad n = 0, 1, 2, \ldots \\ n\frac{\lambda}{2}, & \text{antinodes.} \quad n = 0, 1, 2, \ldots \end{cases}$$

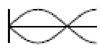
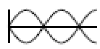
String fixed at both ends:



1. Boundary conditions: $y = 0$ at $x = 0$ and at $x = L$
2. Allowed Freq.: $L = n\frac{\lambda}{2}$, $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$, $n = 1, 2, 3, \ldots$.
3. Fundamental/1[st] harmonics: $\nu_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ 
4. 1[st] overtone/2[nd] harmonics: $\nu_1 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$ 
5. 2[nd] overtone/3[rd] harmonics: $\nu_2 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$ 
6. All harmonics are present.

String fixed at one end:



1. Boundary conditions: $y = 0$ at $x = 0$
2. Allowed Freq.: $L = (2n+1)\frac{\lambda}{4}$, $\nu = \frac{2n+1}{4L}\sqrt{\frac{T}{\mu}}$, $n = 0, 1, 2, \ldots$.
3. Fundamental/1[st] harmonics: $\nu_0 = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$ 
4. 1[st] overtone/3[rd] harmonics: $\nu_1 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$ 
5. 2[nd] overtone/5[th] harmonics: $\nu_2 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$ 
6. Only odd harmonics are present.

Sonometer: $\nu \propto \frac{1}{L}$, $\nu \propto \sqrt{T}$, $\nu \propto \frac{1}{\sqrt{\mu}}$. $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

2.3: Sound Waves

Displacement wave: $s = s_0 \sin\omega(t - x/v)$

Pressure wave: $p = p_0 \cos\omega(t - x/v)$, $p_0 = (B\omega/v)s_0$

Speed of sound waves:

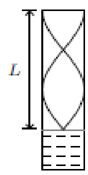
$$v_{\text{liquid}} = \sqrt{\frac{B}{\rho}}, \quad v_{\text{solid}} = \sqrt{\frac{Y}{\rho}}, \quad v_{\text{gas}} = \sqrt{\frac{\gamma P}{\rho}}$$

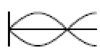
Intensity: $I = \frac{2\pi^2 B}{v}{s_0}^2\nu^2 = \frac{{p_0}^2 v}{2B} = \frac{{p_0}^2}{2\rho v}$

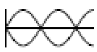
Standing longitudinal waves:

$$p_1 = p_0 \sin\omega(t - x/v), \quad p_2 = p_0 \sin\omega(t + x/v)$$
$$p = p_1 + p_2 = 2p_0 \cos kx \sin\omega t$$

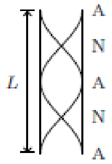
Closed organ pipe:

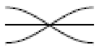
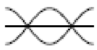
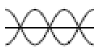


1. Boundary condition: $y = 0$ at $x = 0$
2. Allowed freq.: $L = (2n+1)\frac{\lambda}{4}$, $\nu = (2n+1)\frac{v}{4L}$, $n = 0, 1, 2, \ldots$
3. Fundamental/1[st] harmonics: $\nu_0 = \frac{v}{4L}$ 
4. 1[st] overtone/3[rd] harmonics: $\nu_1 = 3\nu_0 = \frac{3v}{4L}$ 

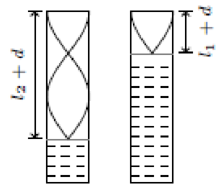
5. 2^{nd} overtone/5^{th} harmonics: $\nu_2 = 5\nu_0 = \frac{5v}{4L}$ 
6. Only odd harmonics are present.

Open organ pipe:



1. Boundary condition: $y = 0$ at $x = 0$
   Allowed freq.: $L = n\frac{\lambda}{2}$, $\nu = n\frac{v}{4L}$, $n = 1, 2, \ldots$
2. Fundamental/1^{st} harmonics: $\nu_0 = \frac{v}{2L}$ 
3. 1^{st} overtone/2^{nd} harmonics: $\nu_1 = 2\nu_0 = \frac{2v}{2L}$ 
4. 2^{nd} overtone/3^{rd} harmonics: $\nu_2 = 3\nu_0 = \frac{3v}{2L}$ 
5. All harmonics are present.

Resonance column:



$$l_1 + d = \frac{\lambda}{2}, \quad l_2 + d = \frac{3\lambda}{4}, \quad v = 2(l_2 - l_1)\nu$$

Beats: two waves of almost equal frequencies $\omega_1 \approx \omega_2$

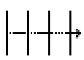
$$p_1 = p_0 \sin\omega_1(t - x/v), \quad p_2 = p_0 \sin\omega_2(t - x/v)$$
$$p = p_1 + p_2 = 2p_0 \cos\Delta\omega(t - x/v)\sin\omega(t - x/v)$$
$$\omega = (\omega_1 + \omega_2)/2, \quad \Delta\omega = \omega_1 - \omega_2 \quad \text{(beats freq.)}$$

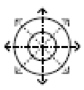
Doppler Effect:

$$\nu = \frac{v + u_o}{v - u_s}\nu_0$$

where, v is the speed of sound in the medium, u_0 is the speed of the observer w.r.t. the medium, considered positive when it moves towards the source and negative when it moves away from the source, and u_s is the speed of the source w.r.t. the medium, considered positive when it moves towards the observer and negative when it moves away from the observer.

2.4: Light Waves

Plane Wave: $E = E_0 \sin\omega(t - \frac{x}{v})$, $I = I_0$ 

Spherical Wave: $E = \frac{aE_0}{r}\sin\omega(t - \frac{r}{v})$, $I = \frac{I_0}{r^2}$ 

Young's double slit experiment

Path difference: $\Delta x = \frac{dy}{D}$

Phase difference: $\delta = \frac{2\pi}{\lambda}\Delta x$

Interference Conditions: for integer n,

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive,} \end{cases}$$

$$\Delta x = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive} \end{cases}$$

Intensity:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta,$$
$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2, \; I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$
$$I_1 = I_2 : I = 4I_0\cos^2\frac{\delta}{2}, \; I_{\max} = 4I_0, \; I_{\min} = 0$$

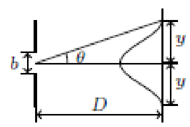
Fringe width: $w = \frac{\lambda D}{d}$

Optical path: $\Delta x' = \mu\Delta x$

Interference of waves transmitted through thin film:

$$\Delta x = 2\mu d = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive.} \end{cases}$$

Diffraction from a single slit:



For Minima: $n\lambda = b\sin\theta \approx b(y/D)$

Resolution: $\sin\theta = \frac{1.22\lambda}{b}$

Law of Malus: $I = I_0\cos^2\theta$